Unit-5

Always remember test statistics part for each topic solving questions. It is the main important thing

DESIGN OF EXPERIMENT

The main objective of design of experiment is either make a high degree of precision or minimizing the error. In design of experiment we are going to design experiment by using different principles of design of experiment which we will discuss later, let first we discuss some terms (or terminology) used in design of experiment.

Terminology in Experimental Design:-

i) **Experiment** → Experiment is means of getting an answer to a question that the experimenter has in mind. Experiment can be divided into two categories; absolute and comparative. Absolute experiment consists of determining the absolute value of some characteristic such as finding correlation coefficient between two variable. While the comparative experiment consists of comparing different types of fertilizers, different varieties of crops etc.

ii) **Treatment** → These are the inputs whose outcomes are to be estimated and compared. These contain different procedures under comparision in experiment. For example in agricultural experiment different types of fertilizers, varieties of crops, cultivation methods etc.

iii) **Experimental Unit** → The smallest division of the experimental area in which the treatments are applied and the effects of treatments are measured is known as experimental unit. For example, in agricultural experiment plot are the experimental unit.

iv) **Yeilds (effects)** → The outcome of the experiment due to the application of treatments in experimental units are called yeilds. In agricultural experiment production of crops on using different fertilizers are yeilds.

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v) **Blocks** → The experimental field is divided into relatively homogenous subgroups which are uniform (similar) among themselves than the field as a whole are called blocks.

vi) **Experimental error** → The variation in experimental measurements made on different experimental units even when they get same treatment is called experimental error. It can be estimated by replication and can be controlled by use of local control.

⊛. Principles of design of experiment:-

According to RA Fisher there are three main principles of design of experiment;

i) **Replication** → The execution of an experiment more than once is called replication. The repeatation of treatment results in more reliable estimate than that of possible with single observation. The main objective of replication is to minimize the error i.e, to maximize the accuracy.

ii) **Randomization** → In randomization the treatments are allocated to various plots in a random manner. This means each treatment will have equal chance of being assigned to an experimental unit. By the use of randomization it reduce the bias of applying a particular treatment to a particular unit.

iii) **Local control** → Local control is used to convert heterogenous field into homogenous by applying local control along row-wise or along column-wise. It reduces the experimental error by making relatively heterogenous experimental materials ~~into~~ relatively homogenous block. So, the experimental material is divided into a number of blocks row-wise or column-wise.

1) Completely Randomised Design (CRD);

row-wise & column-wise दुवैतिर repeat हुने ।

In completely randomised design the two principles are only used which are randomization and replication. In this design, treatments are assigned completely at random manner so each and every experimental unit has equal chance of receiving treatment and it is appropriate for the homogenous experimental material.

Mathematical model:

$$y_{ij} = \mu + \tau_i + e_{ij}\text{, where, } i = 1,2,\ldots,t \text{ (treatment)}; \; j = 1,2,\ldots,r \text{ (replication OR repeatation)}$$

Here,

y_{ij} = j^{th} unit receiving i^{th} treatment.

τ_i = i^{th} treatment effect.

e_{ij} = error due to chance

& μ = Overall mean (OR General mean).

i) Problem to test: (H_0 of treatment)

Null Hypothesis (H_{0T}) → $\mu_A = \mu_B = \mu_C$ (A, B, C are three treatments)

i.e, there is no significance difference between treatment means.

Alternate Hypothesis (H_{1T}) → $\mu_A \neq \mu_B \neq \mu_C$

i.e, There is significance difference between treatment means.

OR At least one treatment mean is different than any other.

ii) Test Statistics (Calculated value):

$$F_{cal} = \frac{\text{MST (i.e, Mean square due to treatment)}}{\text{MSE (i.e, Mean square due to error)}}$$

where, $$\text{MST} = \frac{\text{SST (i.e, Sum of square due to treatment)}}{\text{d.f (i.e, degree of freedom)}}$$

& $$\text{MSE} = \frac{\text{SSE (i.e, Sum of square due to error)}}{\text{d.f (i.e, degree of freedom)}}$$

asked frequently in viva exams
full form asked.

ANOVA table: (i.e, Analysis of Variance table)

Source of variation	d.f.	Sum of Square	Mean sum of square	F_{cal}
Treatment	$(t-1)$	SST	$MST = \frac{SST}{t-1}$ d.f	$F_{cal} = \frac{MST}{MSE}$
Error (Error = total − treatment i.e, $(rt-1)-(t-1)$)	$t(r-1)$	SSE	$MSE = \frac{SSE}{t(r-1)}$ d.f	
Total	$rt-1$ OR $N-1$	TSS		

Here,

$$G = \text{Grand Total}$$

$$\text{Correction Factor } (C.F) = \frac{G^2}{N}$$

total no. of treatments

$$\text{Total sum of Square } (TSS) = \sum_{i=1}^{t} \sum_{j=1}^{r} y_{ij}^2 - C.F$$

each element को square अनि तिनको sum

Now,

$$SSE = TSS - SST$$

iii) **Critical value (tabulated value):-**
The tabulated value of F at α level of significance with $\{t-1, t(r-1)\}$ d.f is $F_\alpha\{t-1, t(r-1)\}$.

iv) **Decision:-** If $F_{tab} > F_{cal}$ then, H_{0T} is accepted otherwise rejected.

v) **Conclusion:-** On the basis of decision we write conclusion as we did before in chapter 2.

Q1. Carry out ANOVA of following output of wheat per field obtained as a result of 3 varieties of wheat A, B, C.

A	10	B	5	A	20	C	15
B	6	A	15	C	11	B	10
C	22	B	12	C	18	A	16

CRD related question since row-wise and column-wise both repeatable

Solution:

Given,

A	10	B	5	A	20	C	15
B	6	A	15	C	11	B	10
C	22	B	12	C	18	A	16

Now, By using deviation (i.e, Subtracting 15 from each value).

We can randomly choose any number. By the subtraction of which we get small numbers

सानो value भएको बेला deviation नगर्दा नि हुन्छ / value ठूलो पर्ने र square गर्दा सजिलो होला भनेर deviation गरेको

A	-5	B	-10	A	5	C	0
B	-9	A	0	C	-4	B	-5
C	7	B	-3	C	3	A	1

Problem to test:

Null Hypothesis (H_{0T}): $\mu_A = \mu_B = \mu_C$

i.e, there is no significance difference between average production on three varieties of wheat.

Alternate Hypothesis (H_{1T}): $\mu_A \neq \mu_B \neq \mu_C$

i.e, there is significance difference between three varieties of wheat.

OR At least one variety of wheat is different than other.

Test Statistics:

We know that,

$$F_{cal} = \frac{MST}{MSE}$$

where, $MST = \frac{SST}{d.f}$

$MSE = \frac{SSE}{d.f}$

ANOVA table

values of A written block-wise (i.e, column-wise)

					T_i
A	-5	0	5	1	1
B	-9	-10	-3	-5	-27
C	7	-4	3	0	6
					$G_T = -20$

ΣA

Grand total

Now, correction factor (C.F) $= \frac{G^2}{N} = \frac{(-20)^2}{12} = \frac{400}{12} = 33.33$

Total Sum of Square (TSS) $= \sum_{i=1}^{t} \sum_{j=1}^{r} y_{ij}^2 - C.F$

$= (-5)^2 + 5^2 + 0 + 1 + (-10)^2 + (-9)^2 + (-5)^2 + (-3)^2 + 0 + (-4)^2 + 7^2 + (3)^2 - 33.33$

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$= 306.67$

we can use T in place of τ

Sum of Square of treatment (SST) $= \dfrac{\Sigma \tau_i^2}{r} - C.F$

$= \dfrac{1^2 + (-27)^2 + 6^2}{4} - 33.33$

$= 158.17$

Now, $SSE = TSS - SST$

$= 306.67 - 158.17$

$= 148.50$

Now, ANOVA table is;

divided by d.f

Source of variation	d.f	SS	MSS	F_{cal}
Treatment	$(t-1) = 3-1 = 2$	$SST = 158.17$	$MST = \dfrac{158.17}{2} = 79.085$	$F_{cal} = \dfrac{MST}{MSE} = \dfrac{79.085}{16.5} = 4.79$
Error	$t(r-1) = 3(4-1) = 9$ OR $11-2 = 9$	$SSE = 148.50$	$MSE = \dfrac{148.5}{9} = 16.5$	
Total	$(rt-1) = 12-1 = 11$	$TSS = 306.67$		

Critical value: The tabulated value of F at 0.05 level of significance with (2,9) d.f is $F_{0.05}(2,9) = 4.26$ — table value of page no 321 at (2,9)

Decision: Since $F_{cal} = 4.79 > F_{tab} = 4.26$. So, H_0 is rejected. i.e, H_1 is accepted.

Conclusion: Hence, there is significance difference between three varieties of wheat.

Q2 The yeild of treatments in different plots are as shown below. Carry out analysis.

t_4 1401	t_3 2536	t_3 2459	t_1 2537	t_3 2827	t_1 2069
t_2 2211	t_1 1797	t_4 1170	t_4 1516	t_4 2104	t_3 2385
t_2 3366	t_1 2104	t_2 2591	t_3 2460	t_4 1077	t_2 2544

Solve (Practice Yourself)

Solving method is same only difference is, 4 blocks were in example 6 blocks in this. Total treatments N will be $6 \times 3 = 18$.

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2> Randomised Block-Design (RBD):- repeatationया row-wise मात्र हुन्छ या column-wise मात्र।

Randomised Block-Design is used for heterogenous experimental materials which is better than CRD. In RBD the treatments are allocated in random manner but randomization (repeatation) is restricted that each treatment must occur once in each row or once in each column. Hence this design is row-wise or column-wise so it is based upon all the principles of design namely replication, randomization and local control.

Mathematical Model:

$$y_{ij} = \mu + \tau_i + \beta_j + e_{ij}$$

This β or Block (B) is additional in this method

where, $i = 1, 2, \ldots, t.$
$j = 1, 2, \ldots, r.$

Here,

y_{ij} = j^{th} block receiving i^{th} treatment.
μ = Overall mean
τ_i = treatment effect
β_j = Effect due to j^{th} block
e_{ij} = Error due to chance.

Problem to test:- दुईवटा hypothesis हुन्छन् यसमा एउटा treatment को लागि र अर्को Block को लागि।

@For treatment

Null Hypothesis(H_{0T}): $\mu_1 = \mu_2 = \ldots = \mu_i$

i.e, There is no significance difference between the treatment means.

Alternate Hypothesis (H_{1T}): $\mu_1 \neq \mu_2 \neq \ldots \neq \mu_i$

i.e, There is significance difference between the treatment means.
OR At least there is one treatment mean different than other.

ⓑ For Block

Null Hypothesis (H_{0B}): $\mu_1 = \mu_2 = \ldots = \mu_j$

i.e, " " (same as above) " "

Alternate Hypothesis (H_{1B}): $\mu_1 \neq \mu_2 \neq \ldots \neq \mu_j$

i.e, " " (same as above) " "

Test Statistics:

ANOVA table:

Source of variation	d.f.	S.S	MSS	F_{cal}
Treatment	$t-1$	SST	$MST = \frac{SST}{d.f}$	$F_T = \frac{MST}{MSE}$
Block	$r-1$	SSB	$MSB = \frac{SSB}{d.f}$	$F_B = \frac{MSB}{MSE}$
Error	Total - treatment - block $(r-1)(t-1)$	SSE	$MSE = \frac{SSE}{d.f}$	
Total	$rt-1$ $= N-1$	TSS		

Here,

$$TSS = SST + SSB + SSE$$

where,

$$TSS = \sum_{i=1}^{t} \sum_{j=1}^{r} y_{ij}^2 - C.F.$$

here, $C.F = \frac{G^2}{N}$

$$SST = \frac{\sum T_{i.}^2}{r} - C.F$$

$$\& \; SSB = \frac{\sum T_{j}^2}{t} - C.F$$

So, $\boxed{SSE = TSS - SSB - SST}$

#We will understand better through following example questions.

Q1. Carry out ANOVA for following design.

A	8	C	10	A	6	B	10
C	12	B	8	B	9	A	8
B	10	A	8	C	10	C	9

Also calculate the relative efficiency of the design with respect to CRD.

Solution: By using deviation (OR Subtracting 8 from each value)

A	0	C	2	A	-2	B	2
C	4	B	0	B	1	A	0
B	2	A	0	C	2	C	1

Problem to test:

ⓐ For treatment:

Null Hypothesis (H_{0T}) : $\mu_A = \mu_B = \mu_C$

i.e, There is no significance difference between treatment means.

Alternate Hypothesis (H_{1T}) : $\mu_A \neq \mu_B \neq \mu_C$

i.e, There is significance difference between treatment means.

OR At least there is one treatment mean which is different than others.

ⓑ For Block:

Null Hypothesis (H_{0B}) : $\mu_1 = \mu_2 = \mu_3 = \mu_4$

i.e, --- " (same as above) " " — —

Alternate Hypothesis (H_{1B}) : $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$

i.e, --- " (same as above) " " — —

Test Statistics:

$$F_T = \frac{MST}{MSE}$$

& $$F_B = \frac{MSB}{MSE}$$

Here, TSS = SST + SSB + SSE

For the calculation of TSS, SST, SSB and SSE ANOVA table is as below;

					T_i
A	0	0	-2	0	-2
B	2	0	1	2	5
C	4	2	2	1	9
T_j	6	2	1	3	G=12

Here,

Grand total (G) = 12

we know that,

Correction Factor (C.F) $= \frac{G^2}{N} = \frac{144}{12} = 12$

$\boxed{TSS = SST + SSB + SSE}$

So, $TSS = \sum_{i=1}^{t}\sum_{j=1}^{r} y_{ij}^2 - C.F$

$= 4 + 4 + 1 + 4 + 16 + 4 + 4 + 1 - 12$

$= 38 - 12$

$= 26$

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$$SST = \sum \frac{T_{i.}^2}{r} - C.F = \frac{(-2)^2 + 5^2 + 9^2}{4} - 12 = 15.5$$

$$SSB = \sum \frac{T_{.j}^2}{t} - C.F = \frac{6^2 + 2^2 + 1^2 + 3^2}{3} - 12 = 4.67$$

Now,

$$SSE = TSS - SSB - SST$$
$$= 26 - 4.67 - 15.5$$
$$= 5.83$$

ANOVA table:

Source of variation	d.f	S.S	MSS	F_{cal}
Treatment	$3-1=2$	$SST = 15.5$	$MST = \frac{15.5}{2} = 7.75$	$F_T = \frac{7.75}{0.4722} = 7.97$
Block	$4-1=3$	$SSB = 4.67$	$MSB = \frac{4.67}{3} = 1.567$	$F_B = \frac{1.567}{0.5722} = 1.61$
Error	6	$SSE = 5.83$	$MSE = \frac{5.83}{6} = 0.9722$	
Total	$12-1=11$	$TSS = 26$		

Critical value

For treatment:

The tabulated value of F at 0.05 level of significance with (2,6) d.f is $F_{0.05}(2,6) = 5.14$

For block

The tabulated value of F at 0.05 level of significance with (3,6) d.f is $F_{0.05}(3,6) = 4.76$

Decision:

For treatment

Since $F_T = 7.97 > F_{tab} = 5.14$. So, H_0 is ~~accepted~~ rejected i.e, H_1 is accepted, i.e, This means there is significance difference between means.

For block.

Since $F_B = 1.61 < F_{tab} = 4.76$. So, H_0 is ~~rejected~~ accepted. i.e, H_1 is rejected.
i.e, There is no significance difference between means.

Now,

$$\text{Precision of RBD / Precision of CRD} = \frac{r(t-1)\,MSE + (r-1)\,MSB}{(rt-1)\,MSE}$$

$$= \frac{4\times2\times0.977 + 3\times1.56}{11\times0.977}$$

$$= 1.164$$

Since, Precision of RBD > 1. So, RBD is more efficient than CRD.

In case $< 1 \Rightarrow$ less effective
$= 1 \Rightarrow$ equally effective

⊛ Missing Plot for RBD:

Let us consider an RBD involving t treatment with r replication each. Let one of the observation say x (i.e, missing value) receiving i^{th} treatment in the j^{th} block is missing.

Let G' = total of all known $rt-1$ values
T' = total of all known values of i^{th} block.
B' = total of all known values of j^{th} block.

Then,

$$\text{missing value } (x) = \frac{T't + B'r - G'}{(t-1)(r-1)}$$

Now we substitute the value of x in place of missing value and carry out analysis as usual.

Because of the change in level of degree of freedom we obtain an upward bias in SST. Hence to get better result subtract an adjustment factor from SST.

i.e, Adjustment factor $(k) = \dfrac{(B' + T't - G')^2}{t(t-1)(r-1)^2}$

Adjusted SST $(SST_A) = SST - k$.

We will understand better with following example:-

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Q. The table given below are yields of 3 varieties in a 4 replicate experiment for which one observation is missing. Estimate the missing value and then analyse the data.

P 19	R 29	P 23	Q 33
Q 26	P ?	Q 27	R 26
R 21	Q 28	R 22	P 26

Solution:

Let the missing value be x. Now we rearrange table as follows:-

Varities	Blocks (replicate) 1st	2nd	3rd	4th	$T_{i.}$
P	19	x	23	26	$T'+x$ $=68+x$
Q	26	28	27	33	114
R	21	29	22	26	98
Total ($T_{.j}$)	66	$B'+x=57+x$	72	85	$G'+x=280+x$

Here, $T'=68$, $B'=57$, $G'=280$, $r=4$, $t=3$

Now, missing value $(x) = \dfrac{T't + B'r - G'}{(r-1)(t-1)} = \dfrac{68\times3+57\times4-280}{3\times2} = 25.33$

Problem to test:

(a) For varieties:- (OR For treatment)

Null Hypothesis (H_{0T}): $\mu_P = \mu_Q = \mu_R$

i.e, There is no significance difference between three varieties.

Alternate Hypothesis (H_{1T}): $\mu_P \neq \mu_Q \neq \mu_R$

i.e, There is significance difference between three varieties.

OR At least one variety is different than other.

(b) For blocks:- (OR For replicates)

Null Hypothesis (H_{0B}): $\mu_1 = \mu_2 = \mu_3 = \mu_4$

i.e, There is no significance difference between four blocks (replicates).

Alternate Hypothesis (H_{1B}): $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$

i.e, There is significance difference between four replicates.

OR At least one replicate is different than other.

Test Statistics:

We know that, $F_T = \dfrac{MST}{MSE}$

$F_B = \dfrac{MSB}{MSE}$

सबैको square को sum

$$TSS = \sum_{i=1}^{t}\sum_{j=1}^{r} y_{ij}^2 - C.F$$

$$= 7927.6 - 7768.86$$

$$= 158.74$$

Correction Factor (CF) $= \dfrac{G^2}{N}$

$$= \frac{(G' + x)^2}{N}$$

$$= \frac{(280 + 25.33)^2}{12}$$

$$= 7768.86$$

$$SST = \frac{\sum_{i=1}^{t} T_{i.}^2}{r} - C.F$$

$$= \frac{(68 + 25.33)^2 + 114^2 + 98^2}{4} - 7768.86$$

$$= 58.76$$

Adjusted treatment of SST (SST_A) = SST − k

Here,

$$k = \frac{(B' + T't - G')^2}{t(t-1)(r-1)^2} = \frac{(57 + 204 - 208)^2}{3 \times 2 \times 9} = 6.68$$

$$\therefore (SST_A) = 58.76 - 6.68 = 52.08$$

$$SSB = \frac{\sum_{j=1}^{r} T_{.j}^2}{t} - C.F = \frac{66^2 + 82.33^2 + 72^2 + 85^2}{3} - 7768.86 = 78.88$$

$$SSE = TSS - SSB - SST_A$$

Now ANOVA table is as follows:

Source of variation	d.f.	Sum of square	Mean sum of square	Fcal
Treatment	3−1 = 2	SST = 52.08	$MST = \frac{52.08}{2} = 26.04$	$F_T = \frac{26.04}{5.55} = 4.69$
Block	4−1 = 3	SSB = 78.88	$MSB = \frac{78.88}{3} = 26.29$	
Error	5	SSE = 27.728	$MSE = \frac{27.728}{5} = 5.55$	$F_B = \frac{26.29}{5.55} = 4.734$
Total	12−2 = 10	TSS = 158.68		

Critical value:

For treatment

Value of F at 0.05 level of significance with (2,5) d.f is $F_{0.05}(2,5) = 5.79$

For Block

Value of F at 0.05 level of significance with (3,5) d.f is $F_{0.05}(3,5) = 5.41$

Decision:

For treatment

Since $F_T = 4.69 < F_{tab} = 5.79$. So, H_0 is accepted i.e, H_1 is rejected.

For block

Since $F_B = 4.734 < F_{tab} = 5.41$. So, H_0 is accepted. i.e, H_1 is rejected.

Conclusion:

There is no significance difference between three varieties.
There is no significance difference between four replicates.

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3.) Latin Square Design (LSD): कतै repeat नहुने

LSD is used for non-homogenous material and it convert the non-homogenous into homogenous. So, LSD is better than RBD that means LSD is more efficient than RBD. Local Control is used in LSD along rowwise and column-wise, which controls error simultaneously. The shap of LSD is square since it contains equal number of rows and columns. It is based upon all the principles of design namely randomization, replication and local control.

Mathematical Model:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \tau_k + e_{ijk}$$

where,
$i = 1,2,3,\ldots,m.$
$j = 1,2,3,\ldots,m.$
$k = 1,2,3,\ldots,m.$

Here,
y_{ijk} = i^{th} row and j^{th} column receiving k^{th} treatment.
μ = constant effect
α_i = effect due to ith row.
β_j = effect due to jth column
τ_k = effect due to k^{th} treatment.
e_{ijk} = error due to chance.

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Problem to test:

(a) For treatment:

Null Hypothesis (H_{0T}): $\mu_1 = \mu_2 = \dots = \mu_m$.

i.e, There is no significance difference between means.

Alternate Hypothesis (H_{1T}): $\mu_1 \neq \mu_2 \neq \dots \neq \mu_m$

i.e, There is significance difference between means

OR At least one mean is different than other.

(b) For rows:-

Similar as for treatment

(c) For columns:-

Similar as for treatment.

We will understand better with example

Test Statistics:

$$F_T = \frac{MST}{MSE} \quad \text{(for treatment)}$$

$$M_C = \frac{MSC}{MSE} \quad \text{(for columns)}$$

$$M_R = \frac{MSR}{MSE} \quad \text{(for rows)}$$

where, $MST = \frac{SST}{d.f}$, $MSC = \frac{SSC}{d.f}$, $MSR = \frac{SSR}{d.f}$ & $MSE = \frac{SSE}{d.f}$

$$\boxed{TSS = SST + SSR + SSC + SSE}$$

Now,

Correction Factor $(C.F) = \frac{G^2}{N}$ where, G = Grand Total.

$$TSS = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} y_{ijk}^2 - C.F$$

$$SST = \frac{\sum T_{..k}^2}{m} - C.F$$

$$SSR = \frac{\sum T_{i..}^2}{m} - C.F$$

$$SSC = \frac{\sum T_{.j.}^2}{m} - C.F$$

Now,

$$\boxed{SSE = TSS - SSR - SSC - SST}$$

ANOVA table is as follows:

Source of Variation	d.f	SS	MSS	F
Treatment	$m-1$	SST	$MST=\frac{SST}{m-1}$	$F_T=\frac{MST}{MSE}$
Row	$m-1$	SSR	$MSR=\frac{SSR}{m-1}$	$F_R=\frac{MSR}{MSE}$
Column	$m-1$	SSC	$MSC=\frac{SSC}{m-1}$	$F_C=\frac{MSC}{MSE}$
Error	$(m-1)(m-2)$	SSE	$MSE=\frac{SSE}{(m-1)(m-2)}$	
Total	m^2-1	TSS		

Q. Set up the analysis of variance for the following results of a design.

A	10	B	15	C	20
B	25	C	10	A	15
C	25	A	20	B	15

Also calculate the efficiency of design over RBD CRD.

Solution:

Taking deviation (i.e, Subtracting 15 from each value).

A	−5	B	0	C	5
B	10	C	−5	A	0
C	10	A	5	B	0

Problem to test:

(a) For treatment:

Null Hypothesis (H_{0T}): $\mu_A = \mu_B = \mu_C$

i.e, There is no ~~sifc~~ significance difference between treatment means.

Alternate Hypothesis (H_{1T}): $\mu_A \neq \mu_B \neq \mu_C$

i.e, There is significance difference between three treatment means

OR. At least one treatment mean is different than other.

(b) For rows:

Null Hypothesis (H_{0R}): $\mu_{1R} = \mu_{2R} = \mu_{3R}$ यो छोडर

i.e, There is no significance difference between rows. योमात्र लेखे पनि हुन्छ।

Alternate Hypothesis (H_{1R}): $\mu_{1R} \neq \mu_{2R} \neq \mu_{3R}$

i.e, There is significant difference between rows.

(c) For columns:

Null Hypothesis (H_{0C}): $\mu_{1C} = \mu_{2C} = \mu_{3C}$

i.e, There is no significance difference between columns.

Alternate Hypothesis (H_{1C}): $\mu_{1C} \neq \mu_{2C} \neq \mu_{3C}$

i.e, There is significance difference between columns.

Test Statistics:

We know that,

$$F_T = \frac{MST}{MSE} \quad \text{(for treatment)}$$

$$F_C = \frac{MSC}{MSE} \quad \text{(for columns)}$$

$$F_R = \frac{MSR}{MSE} \quad \text{(for rows)}$$

and $TSS = SST + SSR + SSC + SSE$

Now,

Row \ Column	1	2	3	T_i
1	-5	0	5	0
2	10	-5	0	5
3	10	5	0	15
T_j	15	0	5	G = 20.

Here,

$$\text{Correction Factor } (C.F) = \frac{G^2}{N} = \frac{G^2}{m^2} = \frac{20^2}{9} = 44.44$$

Now,

$$TSS = \sum_{ijk}^{m} y_{ijk}^2 - CF$$

$$= (-5)^2 + 0^2 + 5^2 + 10^2 + (-5)^2 + 0^2 + 10^2 + 5^2 + 0^2$$

$$= 255.56$$

$$SST = \frac{\sum T_{.k.}^2}{m} - C.F$$

Here,

$$T_{..A} = -5+5+0 = 0$$

$$T_{..B} = 10+0+0 = 10$$

$$T_{..C} = 10-5+5 = 10$$

So, $SST = \frac{0^2+10^2+10^2}{3} - 44.44 = \cancel{38.89}\ 22.226$

sum of rows (i.e, T_i)

$$SSR = \frac{\sum T_{i..}^2}{m} - C.F = \frac{0+5^2+15^2}{3} - 44.44 = 38.89$$

sum of total of columns

$$SSC = \frac{\sum T_{.j.}^2}{m} - C.f = \frac{15^2+0+5^2}{3} - 44.44 = 38.89$$

Now,

$$SSE = TSS - SSR - SSC - SST$$
$$= 256.56 - 38.89 - 38.89 - 22.226$$
$$= 156.56$$

Now, ANOVA table is:-

Source of variation	d.f.	SS	MSS	F
Treatment	$3-1=2$	$SST=22.22$	$MST=\frac{22.22}{2}=11.11$	$F_T=\frac{11.11}{78.28}=0.1414$
Row	$3-1=2$	$SSR=38.89$	$MSR=\frac{38.89}{2}=19.44$	$F_R=\frac{19.44}{78.24}=0.2485$
Column	$3-1=2$	$SSC=38.89$	$MSC=\frac{38.89}{2}=19.44$	$F_C=\frac{19.44}{78.24}=0.2485$
Error	2	$SSE=156.56$	$MSE=\frac{156.56}{2}=78.24$	
Total	$m^2-1=9-1=8$			

Critical value:

For treatment → The value of F at 0.05 level of significance with (2,2) d.f is $F_{0.05}(2,2) = 19.0$

For row → The value of F at 0.05 level of significance with (2,2) d.f is $F_{0.05}(2,2) = 19.0$.

For column → The value of F at 0.05 level of significance with (2,2) d.f is $F_{0.05}(2,2) = 19.0$.

Decision:

For treatment → Since $F_T = 0.1414 < F_{tab} = 19.0$. So, H_0 is accepted.

For row → Since $F_R = 0.2485 < F_{tab} = 19.0$. So, H_0 is accepted.

For column → Since $F_C = 0.2485 < F_{tab} = 19.0$. So, H_0 is accepted.

Conclusion:

There is no significance difference between treatment means.
There is no significance difference between rows.
There is no significance difference between columns.

Again,

i) Efficiency of LSD with relative to RBD $= \dfrac{(m-1)\,MSE + MSC}{m\,MSE}$

$$= \frac{(3-1)\times 78.24 + 19.44}{3\times 78.24}$$

$$= 0.749$$

which is < 1, so LSD is less efficient than RBD.

ii) Efficiency of LSD with relative to CRD $= \dfrac{(m-1)\,MSE + MSR + MSC}{(m+1)\,MSE}$

$$= \frac{(3-1)\times 78.24 + 19.44 + 19.44}{(3+1)\times 78.24}$$

$$= 0.624$$

which is < 1, So, LSD is less efficient than CRD.

Missing plot for LSD:-

Let us consider a $m \times m$ (square) LSD. Let one of the observation x occuring in i^{th} row, j^{th} column and k^{th} treatment is missing.

Let, $G' =$ total of all known m^2-1 values.

$R' =$ total of all known values of i^{th} row.

$C' =$ total of all known values of j^{th} column.

$T' =$ total of all known values of k^{th} treatment.

$$\text{missing value } x = \frac{m(R'+C'+T') - 2G'}{(m-1)(m-2)}$$

We substitute the value in place of missing value x and carry out analysis as usual.

$$\text{Adjustment factor } (k) = \frac{\{(m-1)T' + R' + C' - G'\}^2}{\{(m-1)(m-2)\}^2}$$

$$\text{Adjusted SST } (SST_A) = SST - k.$$

Example. The table given below represents the yeilds of 4 varieties in a 4 replicate experiment for which one observation is missing. Estimate the missing value and then carry out the ANOVA.

A 12	C 19	B 10	D 8
C 18	B 12	D 6	A ?
B 22	D 10	A 5	C 21
D 12	A 7	C 27	B 17.

Solution:

Let missing value be x.

					Total
	A 12	C 19	B 10	D 8	49
	C 18	B 12	D 6	A x	$R'+x = 36+x$
	B 22	D 10	A 5	C 21	58
	D 12	A 7	C 27	B 17	63
Total	64	48	48	$C'+x = 46+x$	$G'+x = 206+x$

Since this is the case of LSD.

So, missing value(x) = $\dfrac{m(R'+C'+T')-2G'}{(m-1)(m-2)}$

Sum of all treatments of A excluding x.

where, $m=4$, $R'=36$, $C'=46$, and $T'=12+5+7=24$

$\therefore x = \dfrac{4(36+46+24)-2\times 206}{(4-1)(4-2)}$

$= 2$

Now,

$G'+x = 206+x = 206+2 = 208 = G$.

Problem to test:

@ For treatment.

Null Hypothesis (H_{0T}):- There is no significance difference between treatments.

Alternate Hypothesis (H_{1T}):- There is significance difference between treatments.

similarly we write for row and column.

Test Statistics:-

we know that, $F_T = \dfrac{MST}{MSE}$ (for treatment)

$F_R = \dfrac{MSR}{MSE}$ (for row)

$F_C = \dfrac{MSC}{MSE}$ (for column).

& $TSS = SSE + SSC + SSR + SST_A$

where, $SST_A = SST -$ adjustment factor (k).

Here, adjustment factor(k) $= \dfrac{[(m-1)T'+R'+C'-G']^2}{\{(m-1)(m-2)\}^2}$

$= \dfrac{[(4-1)\times 24+36+46-206]^2}{\{(4-1)(4-2)\}^2}$

$= 75.11$

Here, Grand Total (G) = 208

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Correction Factor $(C.F) = \frac{G^2}{N} = \frac{(208)^2}{16} = 2704.$

Now

$$TSS = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} y_{ijk}^2 - C.F$$

$$= 12^2+19^2+10^2+8^2+18^2+12^2+6^2+2^2+22^2+10^2+5^2+21^2 + 12^2+7^2+27^2+17^2 - 2704$$

$$= 734$$

$$SSR = \frac{\sum T_{i..}^2}{m} - C.F$$

$$= \frac{(49)^2+38^2+58^2+63^2}{4} - 2704$$

$$= 90.5$$

$$SSC = \frac{\sum T_{.j.}^2}{m} - C.F = \frac{64^2+48^2+48^2+48^2}{4} - 2704 = 48$$

$$SST = \frac{\sum T_{..k}^2}{m} - C.F$$

Here, $T_{..A} = 12+5+7+2 = 26$

$T_{..B} = 22+10+12+17 = 61$

$T_{..C} = 19+18+21+27 = 85$

$T_{..D} = 8+6+10+12 = 36.$

$$\therefore SST = \frac{26^2+61^2+85^2+36^2}{4} - 2704$$

$$= 525.5$$

Now,

$SST_A = SST -$ adjustment factor (k)

$= 525.5 - 75.11$

$= 450.39$

$SSE = TSS - SSC - SSR - SST_A$

$= 734 - 48 - 90.5 - 450.39$

$= 145.11$

Now, ANOVA table is;

consider if any calculation mistakes understand process to solve

Source of variation.	d.f.	SS	MSS	F
Treatment	4-1=3	$SST_A = 450.39$	$MST = \frac{450.39}{3} = 150.13$	$F_T = \frac{150.13}{24.185} = 6.2$
Row	4-1=3	$SSR = 90.5$	$MSR = \frac{90.5}{3} = 30.16$	$F_R = \frac{30.16}{24.185} = 1.24$
Column	4-1=3	$SSC = 48$	$MSC = \frac{48}{3} = 16$	$F_C = \frac{16}{24.185} = 0.66$
Error	6	$SSE = 145.11$	$MSE = \frac{145.11}{6} = 24.185$	
Total	$m^2-1 = 16-1 = 15$	$TSS = 734$		

Critical value:

since same value 3, 6 for all

For treatment, row and column the value of F at 0.05 level of significance and (3,6) degree of freedom is $F_{0.05}(3,6) = 4.76$.

Decision:

for treatment → Since $F_T = 6.2 > F_{tab} = 4.76$. So, H_0 is rejected.

for row → Since $F_R = 1.24 < F_{tab} = 4.76$. So, H_0 is accepted.

for column → Since $F_C = 0.66 < F_{tab} = 4.76$. So, H_0 is accepted.

Conclusion:

There is significance difference between treatments.
There is no significance difference between rows.
There is no significance difference between columns.

⊛. Short Questions:

The questions we did till now are for long [10 marks], there is possibility of short questions [5 marks] also from this unit which are filling up the ANOVA table and taking out analysis. We have did long questions already so it will be quite easier now to know short questions. The following things should be in mind before solving short questions:–

1) Firstly observe the given table i.e, what—what source of variations are provided, generally the questions are of RBD and LSB only.

ⓐ. If treatments, blocks and error are given in source of variation then we should know that it is RBD related question then; we use formulas of RBD in table as we used before for finding values.

ⓑ Similarly if treatments, columns, rows and error are given in source of variation then we should know that it is LSB related question and use formulas in ANOVA table as we used before.

Example 1: Comple the following table for the analysis of variance of a design.

S.V.	d.f.	S.S	MSS	F
Blocks	4 (i.e. t-1)	26.8	?	?
Treatment	3 (i.e. r-1)	?	?	?
Error	?	?	2.5	
Total	?	85.3		

→ Compare this table with ANOVA table for RBD for better understanding

Solution:– The given question is of randomised block design (RBD).

So, Total is given by $= rt-1$ (for defree of freedom)
$= 5 \times 4 - 1$
$= 19$

Rough
$t-1=4$
$\Rightarrow t=5$
& $r-1=3$
$\Rightarrow r=4$

& Error $= 19-4+3$ (for d.f)
$= 12$

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Here,

$$MSE = \frac{SSE}{d.f}$$

$$\text{or, } 2.5 = \frac{SSE}{12}$$

$$\text{or, } SSE = 30.$$

$$SST = TSS - SSB - SSE$$
$$= 85.3 - 26.8 - 30$$
$$= 28.4$$

$$MSB = \frac{SSB}{d.f} = \frac{26.8}{4} = 6.7$$

$$MST = \frac{SST}{d.f} = \frac{28.4}{3} = 9.467$$

$$F_T = \frac{MST}{MSE} = \frac{9.467}{2.5} = 2.786$$

$$\text{\& } F_B = \frac{MSB}{MSE} = \frac{6.7}{2.5} = 2.68$$

Hence the table on felling values now becomes;

S.V.	d.f	SS	MSS	F
Blocks	4	26.8	6.7	2.786
Treatment	3	28.4	9.467	2.68.
Error	12	30	2.5	
Total	19	85.3		

Now, we get ANOVA table and we can now eaisly calculate critical value and make decision and conclusion just same as we did before. It is quite easy.

We will do this only if testing of hypothesis is also asked generally if asks in exams

Similarly we can calculate if LSB table is given.

<u>Note:</u> The most important thing is hence to remember ANOVA table for RBD and LSB and some formulas related to them.